

Name: ANSWER KEY TP: \_\_\_\_\_

### Simplify Complex Numbers

1) Simplify: $\sqrt{-24k^5}$ $\sqrt{-1 \cdot 4 \cdot 6 \cdot k^2 \cdot k^2 \cdot k}$ $i \cdot 2 \cdot k^2 \sqrt{6k}$ $2k^2 i \sqrt{6k}$	2) Simplify: $\sqrt{\frac{-150}{4}}$ $\sqrt{\frac{-1 \cdot 25 \cdot 6}{2^2}}$ $\frac{5i\sqrt{6}}{2}$
3) Simplify: $-4 - (-8 + 6i) - 2 + 5i$ $-4 + 8 - 6i - 2 + 5i$ $2 - i$	4) Simplify: $(-2i) + 2 + (-6 + i)$ $-2i + 2 - 6 + i$ A) $-6 - i$ B) $4 - i$ C) $-4 - 3i$ D) $-4 - i$
5) $-\sqrt{96}$ is equivalent to: a) $-4i\sqrt{6}$ b) $4i\sqrt{6}$ c) $-4\sqrt{6}$ d) $4\sqrt{6}$ $-\sqrt{16 \cdot 6}$ $-4\sqrt{6}$	6) $\sqrt{75xy} \cdot \sqrt{-2x^3}$ $\sqrt{25 \cdot 3 \cdot x \cdot y} \cdot \sqrt{-1 \cdot 2 \cdot x^2}$ $5x^2 i \sqrt{6y}$
7) Simplify: $-8i^2 - 3i + 8 - 2i^2$ $8 - 3i + 8 + 2$ $18 - 3i$	8) Simplify: $3\sqrt{-196} + 3\sqrt{-64}$ $3\sqrt{-1 \cdot 196} + 3\sqrt{-1 \cdot 64}$ $42i + 24i = 66i$

### Multiply Complex Numbers

9) Simplify: $-7i(5 + 5i)$ $-35i - 35i^2$ $35 - 35i$	10) Simplify: $(7 + 4i)(3 + 5i)$ $21 + 35i + 12i + 20i^2$ $1 + 47i$
11) Simplify: $-5i \cdot -7i(-5 + 3i)$ $-5i(35i - 21i^2)$ $-175i^2 + 105i$ A) $-175 - 105i$ B) $140 - 105i$ C) $175 - 105i$ D) $-120 - 200i$	12) $-6i \cdot -4i$ $24i^2 = -24$ A) $-24$ B) $20i$ C) $24$ D) $32i$
13) Simplify: $(-2 - 7i)^2$ $(-2 - 7i)(-2 - 7i)$ $4 + 14i + 14i + 49i^2$ $-45 + 28i$	14) Simplify: $-3i \cdot 8i(-5 - 4i)$ $-24i^2(-5 - 4i)$ $120i^2 + 96i^2$ $-120 - 96i$

PUSH IT TO THE LIMIT.

<p>15) Find values of <math>x</math> and <math>y</math> to make each equation true.</p> $3x + 2iy = 6 + 10i$ <p><math>x = 2</math> <math>y = 5</math></p>	<p>16) Simplify:</p> $i^5 \cdot 5i^7 \cdot -8i^5 \cdot -i^3$ $i^5 \cdot i^7 \cdot -8 \cdot i^5 \cdot -i^3$ $40 \cdot i^{20} = 40(i^4)^5 = \boxed{40}$
<p>17) The voltage <math>E</math>, current <math>I</math>, and impedance <math>Z</math> in a circuit are related by <math>E = I \cdot Z</math>. Find the voltage (in volts) in each of the following circuits given the current and impedance.</p> <p><math>I = -5 - 9j</math> amps, <math>Z = 3 + 4j</math> ohms</p> $(-5 - 9j)(3 + 4j) = -15 - 20j - 27j - 36j^2$ $= -15 - 47j + 36$ $\boxed{21 - 47j}$	<p>18) Which of the following is equivalent to <math>i^{46}</math>?</p> <p><math>(i^4)^{11} (i^2)</math> <math>-1</math></p> <p>A. 1 B. <math>i</math> C. <math>-1</math> D. <math>-i</math></p>

### Radical and Complex Conjugates

<p>19) Simplify:</p> $\frac{2}{8\sqrt{10} - \sqrt{7}} \cdot \frac{8\sqrt{10} + \sqrt{7}}{8\sqrt{10} + \sqrt{7}} = \frac{16\sqrt{10} + 2\sqrt{7}}{640 - 7}$ $\boxed{\frac{16\sqrt{10} + 2\sqrt{7}}{633}}$	<p>20) Simplify:</p> $\frac{-1 - 8\sqrt{3}}{7 + 6\sqrt{10}} \cdot \frac{7 - 6\sqrt{10}}{7 - 6\sqrt{10}} = \frac{-7 + 6\sqrt{10} - 56\sqrt{3} + 48\sqrt{30}}{49 - 360}$ $\frac{-7 + 6\sqrt{10} + 56\sqrt{3} + 48\sqrt{30}}{-311}$
<p>21) Simplify:</p> $\frac{6n^2}{2\sqrt{2n^3} - 5} \cdot \frac{2\sqrt{2n^3} + 5}{2\sqrt{2n^3} + 5}$ $\frac{12n^3\sqrt{2n} + 30n^2}{4n^3 - 25}$ $\frac{12n^3\sqrt{2n} + 30n^2}{4n^3 - 25}$	<p>22) Simplify:</p> $\frac{4i}{-4 + 2i} \cdot \frac{-4 - 2i}{-4 - 2i} = \frac{-16i - 8i^2}{16 - 4i^2} = \frac{8 - 16i}{20}$ $\boxed{\frac{2 - 8i}{5}}$
<p>23) Simplify:</p> $\frac{3 - 9i}{3 + 8i} \cdot \frac{3 - 8i}{3 - 8i} = \frac{9 - 27i - 27i + 72i^2}{9 - 64i^2}$ $\frac{-63 - 54i}{73}$ $\boxed{\frac{-63 - 54i}{73}}$	<p>24) Write the expression as a complex number in standard form.</p> $\frac{a + bi}{c - di} \cdot \frac{c + di}{c + di} = \frac{ac + adi + bci + bdi^2}{c^2 - d^2i^2}$ $\frac{ac + adi + bci - bd}{c^2 + d^2}$
<p>25) Simplify:</p> $\frac{3}{2 - 4i} - (3 + 2i)$ $\frac{3}{2 - 4i} \cdot \frac{2 + 4i}{2 + 4i} = \frac{6 + 12i}{4 + 16}$ $\frac{6 + 12i}{20} - \frac{20(3 + 2i)}{20}$ $\frac{6 + 12i - 60 - 40i}{20}$ $\frac{-54 - 28i}{20} \rightarrow \boxed{\frac{-27 - 14i}{10}}$	

**PUSH IT TO THE LIMIT.**



## Number Properties

26) What is the maximum value of  $2a$  for  $a$  and  $b$  satisfying the system of inequalities below?

$$a \geq 0$$

$$b \geq 0$$

$$a + b \leq 8$$

$$a = 0 \\ b = 8$$

- A. 1  
B. 2  
C. 8  
D. 16  
E. Cannot be determined from the given information

Trial 1:

$$a = 0 \quad b = 8 \\ 2(0) = 0$$

Trial 2:

$$a = 8 \quad b = 0 \\ 2(8) = 16$$

Pattern/Explanation:

you will need to find the max value for  $a$ , minimize  $b$ .

Trial 3:

Trial 4:

27) Which of the following is true for all consecutive integers  $m$  and  $n$  such that  $m < n$ ? \*consecutive means numbers in order

- A.  $m$  is odd  $\times$   
B.  $n$  is odd  $\times$   
C.  $n - m$  is even  $\times$   
D.  $n^2 - m^2$  is odd  $\checkmark$   
E.  $m^2 + n^2$  is even  $\times$

$$m < n \quad C. \quad m = 2 \quad n = 3 \\ 3 - 2 = 1$$

$$d. \quad 3^2 - 2^2 = 9 - 4 = 5 \\ 4^2 - 3^2 = 16 - 9 = 7$$

$$e. \quad 2^2 + 3^2 = 4 + 9 = 13 \\ 3^2 + 4^2 = 9 + 16 = 25$$

## Geometric Formulas

28)

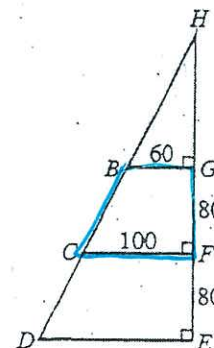
The volume,  $V$ , of a sphere is determined by the formula  $V = \frac{4\pi r^3}{3}$ , where  $r$  is the radius of the sphere. What is the volume, in cubic centimeters, of a sphere with a diameter 6 centimeters long?

- F.  $36\pi$   
G.  $72\pi$   
H.  $108\pi$   
J.  $144\pi$   
K.  $288\pi$

$$d = 6 \\ r = 3 \\ V = \frac{4\pi \cdot 3^3}{3} \\ = 36\pi$$

29)

In the figure below,  $B$  and  $C$  are on  $\overline{HD}$  and  $G$  and  $F$  are on  $\overline{HE}$ . The measurements given are in inches. Both  $BGFC$  and  $CFED$  are trapezoids. The area,  $A$ , of a trapezoid is given by  $A = \frac{1}{2}h(b_1 + b_2)$ , where  $h$  is the height and  $b_1$  and  $b_2$  are the lengths of the 2 parallel sides.



$$= \frac{1}{2} 80 (60 + 100) \\ = 6400$$

39. What is the area of  $BGFC$ , in square inches?

- A. 2,500  
B. 5,400  
C. 6,400  
D. 7,000  
E. 12,800

30)

9. A formula for the volume,  $V$ , of a right circular cylinder in terms of its radius,  $r$ , and its height,  $h$ , is  $V = \pi r^2 h$ . What is the height, in centimeters, of a right circular cylinder that has a volume of  $270\pi$  cubic centimeters and a radius of 3 centimeters?

- A. 30  
B. 90  
C. 279  
D. 810  
E. 2,430

$$270\pi = \pi \cdot 3^2 \cdot h$$

$$h = 30$$

31)

1. The total surface area, including top and bottom, of a right circular cylinder is given by the expression  $2\pi rh + 2\pi r^2$ , where  $r$  is the radius and  $h$  is the height of the cylinder. What is the total surface area, in square inches, of a cylinder that is 8 inches high and has a diameter of 6 inches?

- F.  $24\pi$   
G.  $33\pi$   
H.  $60\pi$   
J.  $66\pi$   
K.  $168\pi$

$$d = 6 \quad r = 3$$

$$2\pi \cdot 3 \cdot 8 + 2 \cdot \pi \cdot 3^2$$

$$= 48\pi + 18\pi$$

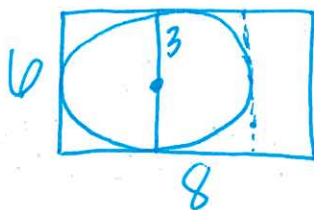
$$= 66\pi$$

## Complex Area:

32)

- What is the area, in square centimeters, of the largest circle that can fit within a rectangle measuring 6 centimeters by 8 centimeters?

- A.  $6\pi$   
B.  $8\pi$   
C.  $9\pi$   
D.  $16\pi$   
E.  $48\pi$

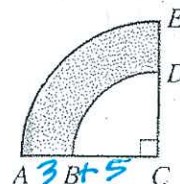


$$= \pi r^2$$

$$= 9\pi$$

33)

- The diagram below shows a quarter of each of 2 circles both having point  $C$  as their center. Point  $B$  lies on  $\overline{AC}$  and point  $D$  lies on  $\overline{CE}$ . The length of  $\overline{BC}$  is 5 inches and the length of  $\overline{AB}$  is 3 inches. What is the area, in square inches, of the shaded portion?



- F.  $4\pi$

- G.  $\frac{25}{4}\pi$

- H.  $\frac{39}{4}\pi$

- J.  $\frac{55}{4}\pi$

- K.  $16\pi$

$$\text{shaded: } \frac{64\pi}{4} - \frac{25\pi}{4} = \frac{39\pi}{4}$$

$$\text{unshaded: } \frac{25\pi}{4}$$

- 34) Find the area of the figure below. Leave your answer in terms of pi.

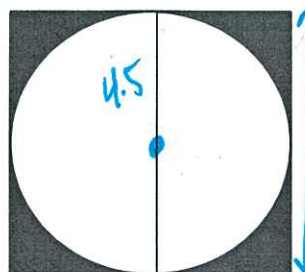


$$36\pi$$

$$300 + 36\pi$$

$$\approx 413.1 \text{ ft}^2$$

- 35) The figure below is a square with an inscribed circle whose edges touch each side of the square. If one side of the square is 9 cm, what is the area of the shaded region?



$$A_0 = 9^2$$

$$= 81$$

$$A_0 = 20.25\pi$$

$$\approx 63.6$$

$$81 - 63.6$$

$$= 17.4$$